MATHEMATICS SPECIALIST UNIT 1

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	96	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed   **65% (96 Marks)**

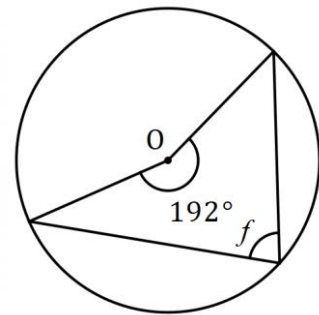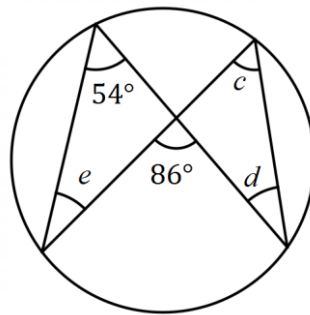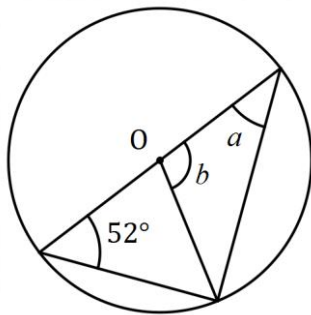This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9   **(6 marks)**

Determine the size of the angles marked a, b, c, d, e and f shown in the circles below. Where marked, O is the centre of the circle.

Question 10   **(5 marks)**

Three forces act on an object so that it remains in equilibrium. Two of the forces have magnitudes of 80 N and 110 N and the angle between their directions is 105°. Determine the magnitude of the third force and the angle its direction makes with the smaller force.

Question 11   **(9 marks)**

(a)   An art gallery plans to display a single painting on each of the three walls in a room.
        Determine how many arrangements of paintings are possible in the room if they have a
        selection of 24 different paintings to choose from.   (2 marks)

(b)   In another room, the gallery plan to hang 8 different paintings in a row. If 2 of the paintings
        are by the artist McGrath, determine the number of different arrangements of paintings
        that are possible when

        (i)   the paintings by McGrath must be at both ends.   (2 marks)

        (ii)   the paintings by McGrath must be next to each other.   (2 marks)

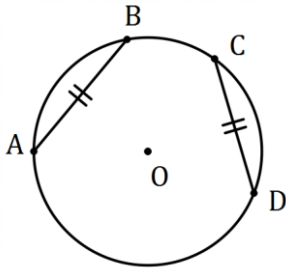   (iii)   the paintings by McGrath must be apart and neither of them at the ends. (3 marks)

Question 12 (8 marks)

(a) Prove that chords of equal length subtend equal angles at the centre of a circle.

(3 marks)

(b) Points P and Q lie on a circle of radius 23.3 cm so that $PQ = 21$ cm. Determine

(i) the distance of chord PQ from the centre of the circle. (3 marks)

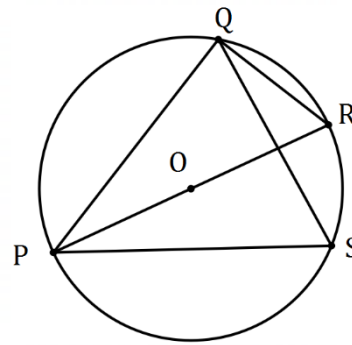(ii) the angle subtended by chord PQ at the centre of the circle. (2 marks)

See next page

Question 13   **(7 marks)**

(a)   The diagram shows points P, Q, R and S that
      lie on the circumference of a circle centre O.
      PR is a diameter and the size of $\angle QPR = 27°$.
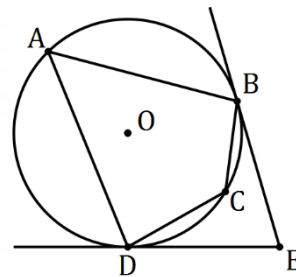
      Determine, with reasons, the size of $\angle PSQ$.   (3 marks)

(b)   In the diagram shown, A, B, C and D are points
      on the circumference of a circle with centre O.
      Tangents to the circle at B and D intersect at E.

      Determine, with justification, the size of $\angle BCD$
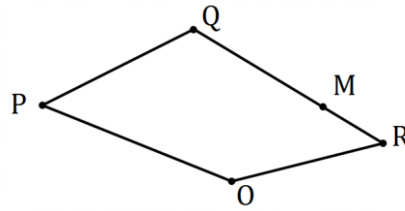   when $\angle BED = 72°$.   (4 marks)

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Question 14 (8 marks)

In quadrilateral $OPQR$ shown below, M lies on QR so that $|\overrightarrow{QM}| = 3|\overrightarrow{MR}|$.

(a) If $\overrightarrow{OP} = \mathbf{p}, \overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{OR} = \mathbf{r}$, express the following in terms of $\mathbf{p}, \mathbf{q}$ and/or $\mathbf{r}$.

(i) $\overrightarrow{PR}$. (1 mark)

(ii) $\overrightarrow{RM}$. (2 marks)
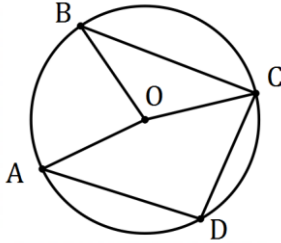
(iii) $\overrightarrow{PM}$. (2 marks)

(b) If O is the origin and points P, Q and R have coordinates $(-2, 39), (28, -14)$ and $(32, -18)$ respectively, determine the distance PM. (3 marks)

Question 15 (8 marks)

(a)   The vertices of quadrilateral $ABCD$ lie on the circumference of a circle centre O shown below. Given that $\angle ADC = 95°$ and $\angle AOB = 84°$, determine with reasoning the size of angle BCO.   (4 marks)

(b)   The vertices of triangle ABC lie on the circumference of a circle. Given that $AB = 10$ cm, $AC = 7$ cm and $BC = 6$ cm, prove by contradiction that AB is not a diameter of the circle.   (4 marks)

Question 16   **(7 marks)**

(a)   A calculator can generate random integers, 10 to 25 inclusive. Use the pigeonhole
       principle to explain why 49 random integers should be generated to be certain that at least
       4 of them are the same.   (3 marks)

(b)   16 customers bought a total of 130 items from a supermarket. Given that each customer
       bought at least one item, show that at least two of the customers bought the same number
       of items.   (4 marks)

See next page

Question 17   **(9 marks)**

(a)   Determine the scalar product of

   (i) $3.5\mathbf{i} + 6.5\mathbf{j}$ and $8\mathbf{i} - 2\mathbf{j}$.   (1 mark)

   (ii)   two vectors with directions 60° apart that have magnitudes of 15 and 18.   (1 mark)

(b)   Given that $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 7$ simplify $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) + \mathbf{a} \cdot (\mathbf{a} - 2\mathbf{b})$.   (3 marks)

(c)   The position vectors of points P, Q and R are $\begin{pmatrix} 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$. Show use of a vector
       method to determine the size of angle PQR.   (4 marks)

See next page

Question 18   **(8 marks)**

The Trinity school yearbook is produced by a committee of 3 teachers and 8 students. 5 teachers and 17 students have nominated for the committee.

(a)   Determine how many different committees could be formed from the nominations.

(2 marks)

(b)   The student nominations include two sets of twins. Determine how many different committees could be chosen that include at least one set of twins.   (4 marks)

(c)   Suppose one of the teachers in the committee will be appointed as treasurer and one of the students will be appointed as secretary. Determine how many different committees can be formed with this structure.   (2 marks)

Question 19 **(8 marks)**

Oil platform T lies 66.5 km away from another oil platform F on a bearing of 215°. A steady current of 4.5 km per hour flows between the platforms on a bearing of 110°. Flynn has a small boat at F, with a cruising speed of 12 km per hour, he needs to arrive at T by 4 pm.
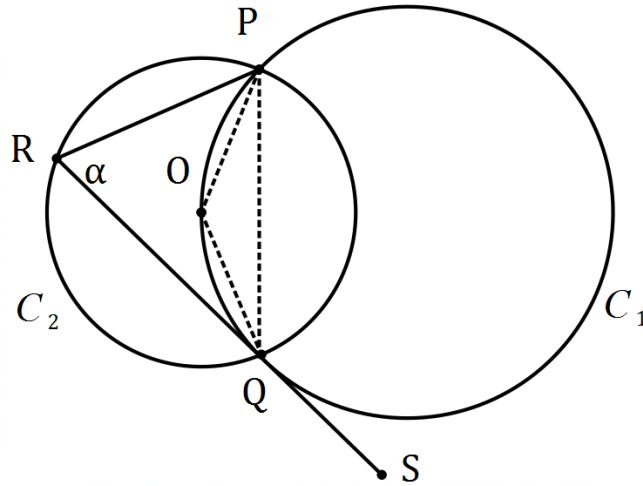
Determine the bearing that Flynn should steer the boat and the latest time he should depart from platform F.

Question 20 (5 marks)

The diagram below shows Circles C_1 and C_2 intersecting at points P and Q. C_1 passes through O, the centre of C_2. R lies on C_2 so that line segment RS is tangential to C_1 at Q. $\angle PRQ = \alpha$.

(a) Determine $\angle POQ$ in terms of α. (1 mark)

(b) Explain why $\angle PQS = 2\alpha$. (1 mark)

(c) Prove that $PQ = QR$. (3 marks)

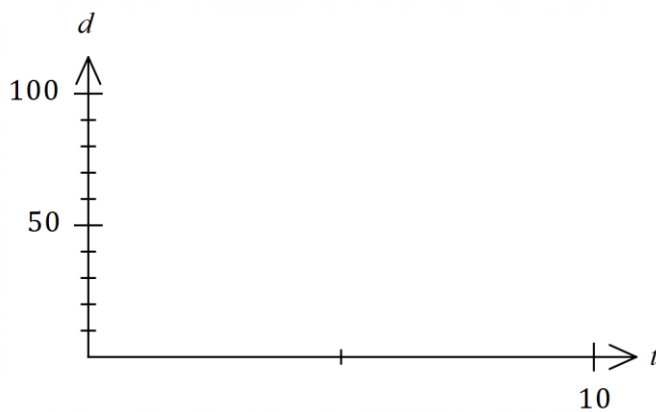**Question 21**   **(8 marks)**

Particle A, initially at the point with position vector $42\mathbf{i} - 25\mathbf{j}$ cm, moves with a constant velocity of $-8\mathbf{i} + 15\mathbf{j}$ cm/s. Particle B is stationary at the point with position vector $-35\mathbf{i} + 11\mathbf{j}$.

(a)   Determine the initial distance of A from B.   (2 marks)

(b)   Determine an expression for the distance d between A and B after t seconds.   (3 marks)

(c)   Sketch a graph of d against t and hence determine the time that minimises d and state what this minimum distance is.   (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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